

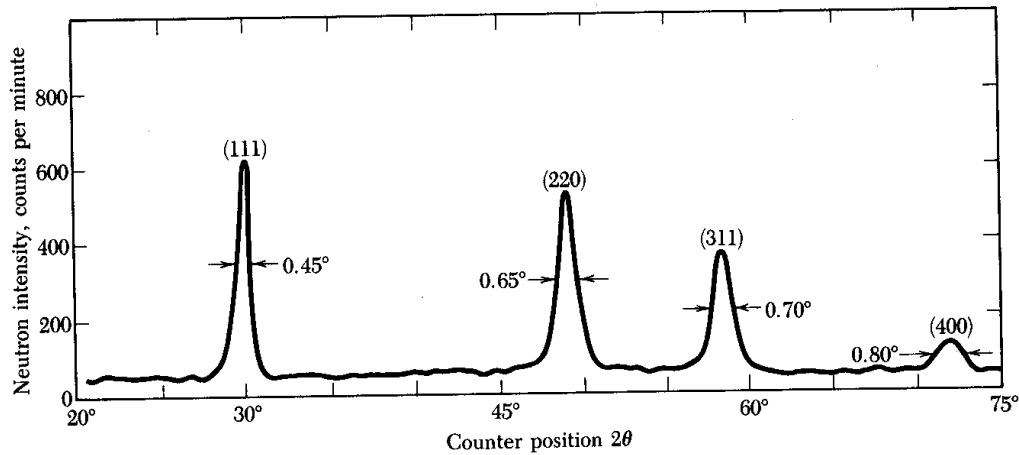
### Work Sheet # 3

1. Find the distribution of atoms in the atomic planes (010), (110) and (111) of a simple cubic crystal structure.
2. Prove that the lattice planes with the greatest densities of points are the {111} planes in a face-centered cubic Bravais lattice and the {110} planes in a body-centered cubic Bravais lattice.  
[Hint: (1) This is most easily done by exploiting the relation between families of lattice planes and reciprocal lattice vectors. (2) Take the primitive vectors for fcc lattice as:  $\vec{a}_1 = \frac{a}{2}(\hat{y} + \hat{z})$ ,  $\vec{a}_2 = \frac{a}{2}(\hat{x} + \hat{z})$  and  $\vec{a}_3 = \frac{a}{2}(\hat{y} + \hat{x})$ ].
3. Show that the expression  $d(hkl) = \frac{2\pi}{|\vec{G}|}$  can be rewritten as  $2 d \sin \theta = n\lambda$ , where  $\theta$  is the angle between the incident beam of x-radiation and the crystal plane.
4. Find the structure factor for an fcc crystal structure when the scattering peaks are indexed to a simple cubic lattice.
5. The number density of the hydrogen atom in its ground state is

given by  $n(r) = \frac{e^{-2r/a_0}}{\pi a_0^3}$ , where  $a_0$  is the Bohr radius. Show that the form

factor is  $f_G = \frac{16}{(4 + G^2 a_0^2)^2}$ .

6. The neutron diffraction pattern for powdered diamond is in figure shown. Find:
- The structure factor of the basis of the conventional cell of diamond.
  - Show that the allowed reflections of the diamond structure satisfy  $h + k + \ell = 4n$ , where all indices are even and  $n$  is an integer, or else all indices are odd.
  - The zeros of structure factor.



7. Consider a simple cubic crystal structure with a cube edge  $a = 3.5 \text{ \AA}$  and suppose it is used to scatter  $3.1 \text{ \AA}$  x-rays.
- Find all sets of planes that satisfy the Bragg condition and for each peak.
  - Find the Bragg angle.

The followings are MCQ's. Please choose the correct answer:

8. If the wave function of the hydrogen atom in its ground state is given by  $\psi(r) = \frac{e^{-r/a_0}}{a_0 \sqrt{\pi a_0}}$ , (where  $a_0$  is the Bohr radius) the number density of the hydrogen atom must have the form

a.  $n(r) = \frac{e^{-r/a_0}}{a_0 \sqrt{\pi a_0}}$       b.  $n(r) = \frac{e^{-r/2a_0}}{2\pi a_0^2}$       c.  $n(r) = \frac{e^{r/2a_0}}{2\pi a_0^2}$

d.  $n(r) = \frac{e^{2r/a_0}}{\pi a_0^3}$       e.  $n(r) = \frac{e^{-2r/a_0}}{\pi a_0^3}$ .

9. Given two identical atoms of the body-centered-cubic (bcc) basis which are located at  $(x_1, y_1, z_1) \equiv (0, 0, 0)$  and  $(x_2, y_2, z_2) \equiv (1/2, 1/2, 1/2)$ . If the structure factor  $S_G$  can be expressed in terms of the form factor of an atom,  $f$ , as  $S_G(hkl) = f(1 + e^{-i\pi(h+k+\ell)})$ , then the peak intensities that can be observed by x-ray diffraction, are for the planes:

- a. (100) and (311)
- b. (200) and (110)
- c. (221) and (100)
- d. (111) and (210)
- e. (111) and (414)

10. The intensity peaks of the face-centered-cubic and body-centered-cubic structures are usually indexed using

- a. special kind of vectors called reciprocal vectors of first kind.
- b. the conjugate of reciprocal vectors.
- c. complex cubic lattice vectors.
- d. simple cubic lattice vectors.
- e. special kind of vectors called reciprocal vectors of second kind.